The Chain Rule

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Use the Chain Rule when we need to take the derivative of a composite function.

f(g(x))

Chain Rule

$$\frac{d}{dx}\left[f(g(x))\right] = f'\left(g(x)\right)g'(x)$$

the derivative of the outer function times

the derivative of the inner function.

$$y=(x^2+1)^3$$

outer

$$(u)^3$$

Inner

$$x^2+1$$

$$3(u)^2 \cdot 2x$$

$$6x(x^2+1)^2$$

$$y = (3x-2x^2)^3$$

$$y' = 3(3x-2x^2)^2 \cdot (3-4x)$$

outer inner

$$y' = \boxed{3(3x-2x^2)^2(3-4x)}$$

$$f(x)=\sqrt[3]{\left(x^2-1\right)^2}$$

$$(x^2-1)^{\frac{2}{3}}$$

$$\frac{2}{3}(x^2-1)^{-\frac{1}{3}}\cdot 2x$$

$$\frac{4x}{3(x^2-1)^{\frac{1}{3}}}=\frac{4x}{3\sqrt[3]{x^2-1}}$$

$$f(x) = \frac{-7}{(2t-3)^2}$$

$$-7(2t-3)^{-2}$$

$$-7\left[-2(2t-3)^{-3} \cdot (2)\right]$$

$$28(2t-3)^{-3}$$

$$y' = \frac{28}{(2t-3)^3}$$

$y = \sin(2x)$

$y' = \cos(2x) \cdot 2$

$y' = 2\cos(2x)$

$y = \cos(x-1)$

$-\sin(x-1)$

y= tan (3x)

$3\sec^2(3x)$

$$y=\sqrt{\cos x}$$

$$y = x^2\sqrt{1-x^2}$$

$$x^2(1-x^2)^{\frac{1}{2}}$$

Product Rule:

$$2x(1-x^2)^{\frac{1}{2}} + x^2\left[\frac{1}{2}(1-x^2)^{-\frac{1}{2}} \cdot (-2x)\right]$$

Chain Rule

$$2x(1-x^2)^{\frac{1}{2}} - x^3(1-x^2)^{-\frac{1}{2}}$$

$$x(1-x^2)^{-\frac{1}{2}}\left[2(1-x^2) - x^2\right]$$

$$2-2x^2-x^2$$

$$\frac{x(2-3x^2)}{\sqrt{1-x^2}}$$

$$f(x)=\frac{x}{\sqrt[3]{x^2+4}}$$

$$\frac{x}{(x^2+4)^{\frac{1}{3}}}$$

$$\frac{1\cdot(x^2+4)^{\frac{1}{3}}-\left[\underbrace{x\left(\frac{1}{3}(x^2+4)^{-\frac{2}{3}}\cdot(2x)\right)}_{2x^2}\right]}{(x^2+4)^{\frac{2}{3}}}$$

$$\frac{(x^2+4)^{\frac{1}{3}}-\frac{2}{3}x^2(x^2+4)^{-\frac{2}{3}}}{(x^2+4)^{2/3}}$$

$$\frac{(x^2+4)^{-2/3}\left[(x^2+4)-\frac{2}{3}x^2\right]}{(x^2+4)^{2/3}}$$

$$\frac{\frac{1}{3}x^2+4}{(x^2+4)^{2/3}(x^2+4)^{2/3}}$$

$$\frac{\frac{1}{3}x^2+4}{(x^2+4)^{4/3}}=\frac{1}{3}\,\frac{(x^2+12)}{(x^2+4)^{4/3}} \qquad \frac{x^2+12}{3\sqrt[3]{(x^2+4)^4}}$$

$$\frac{x^2+12}{3\sqrt[3]{(x^2+4)^4}} \qquad \frac{1}{3}\cdot\frac{x^2+12}{(x^2+4)^{4/3}}$$

$$x^{\frac{1}{3}}$$

$$x^{-\frac{2}{3}} \quad x$$

$$-\frac{2}{3}+\cancel{x}=\frac{1}{3}+\frac{2}{3}$$

$$y=\left(\frac{3x-1}{x^2+3}\right)^2$$

$$2\left(\frac{3x-1}{x^2+3}\right)$$

$f(t)=\sin^3(4t)$

$(\sin(4t))^3$

Outside

$3(\sin(4t))^2 \cdot$

Inside

$\frac{d}{dx}\sin(4t)$

$3(\sin(4t))^2 \cdot \cos 4t \cdot 4$

$12\sin^2(4t)\cos(4t)$

Find an equation of the tangent line to the graph of $f(x)= 2\sin x +\cos 2x$ at the point

$(\pi, 1)$. Then find all of the values of x in the interval $(0, 2\pi)$ at which the graph of f has a horizontal tangent.

$f(x) = 2\sin x + \cos 2x$

$2\cos x + (-\sin(2x)\cdot 2)$ derivative

Chain Rule

$\boxed{2\cos x - 2\sin(2x)}$ $(\pi, 1)$

$2\cos(\pi) - 2\sin(2\pi)$ find slope

$\boxed{= -2 \text{ (slope)}}$

$y - 1 = -2(x - \pi)$ pt-slope

$y - \cancel{1} = -2x + 2\pi$

$+1$

$\boxed{y = 1 - 2x + 2\pi}$ eq of tangent line $(\pi, 1)$

Horizontal Tangent lines

$2\cos x - 2\sin 2x = 0$

$\frac{2\cos x}{2} = \frac{2\sin 2x}{2}$

$\cos x = \sin 2x$

$\cos\frac{\pi}{6} = \sin 2\left(\frac{\pi}{6}\right)$

horiz. tan line at

$\frac{\pi}{}$